Predicate Logic

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Warm up

1. Discuss with your neighbors whether you plan on watching the superbowl. If you are what's the BEST superbowl snack?

2. Is the following argument valid?

$$\begin{array}{l} p \vee q \\ \neg p \vee r \\ \hline \therefore\ q \vee r \end{array}$$

$((p \vee q) \wedge (\neg p \vee r)) \to (q \vee r)$

CNF, wow it sucks....

$\neg((p \vee q) \wedge (\neg p \vee r)) \vee (q \vee r)$

$(\neg(p \vee q) \vee \neg(\neg p \vee r)) \vee (q \vee r)$

$((\neg p \wedge \neg q) \vee (p \wedge \neg r)) \vee (q \vee r)$

$(((\neg p \wedge \neg q) \vee p) \wedge ((\neg p \wedge \neg q) \vee \neg r)) \vee (q \vee r)$

$(((\neg p \vee p) \wedge (\neg q \vee p)) \wedge (\neg p \vee \neg r) \wedge (\neg q \vee \neg r)) \vee (q \vee r)$

$(\underbrace{\neg p \vee p \vee q \vee r}_{T}) \wedge (\underbrace{\neg q \vee p \vee q \vee r}_{T}) \wedge (\underbrace{\neg p \vee \neg r \vee q \vee r}_{T}) \wedge (\underbrace{\neg q \vee \neg r \vee q \vee r}_{T})$

T

:)

Logisitics

- codelet 2 due tomorrow
- lab 2 due <u>next</u> Friday
- grades posted for
  - → lab 01
  - → codelet 01

overall people did well for the lab I tried to give feedback on content and style

Course Goals

- more precise in thinking & arguing
- improved coding skills

p Every faculty member hates snow days

q Prof. Forrest hates snow days

```
if x > 3:
    print(hi)
```

$p := x == 3$

$x == 1$

$P(x)$

predicate

P

a single variable

$P(x) := x > 3$

Domain?

3 2

$P(3)$ $3 > 3$

$P(2)$ $2 > 3$

$P(x)$ s.t. $x \in \mathbb{N}$: $x > 3$

$P(x) := x+1 > 2 \quad x \in \{3, 4, 5\}$

$P(3)$, $P(4)$, $P(5)$

$P(x)$ is true

$\forall$ "for all" universal quantification

"for every"

$\exists$: "there exists"

"there is at least one"

existential quantification

(1) $\forall p \in P : At(p, Colgate) \rightarrow lovesCats(p)$

where P is the set of people

$$P := \{p_1, p_2, p_3, \ldots, p_n\}$$

(2) $\forall p \in P : At(p, Colgate) \wedge lovesCats(p)$

Are (1) and (2) the same?

$\exists p \in P : At(p, Syracuse) \land lovesCats(p)$

$\exists p \in P : At(p, Syracuse) \to lovesCats(p)$

more permissive

no person lives in Syracuse

loves (x, y)

FigPetter (x)

Best person := loves$(x$, FigPetter$(y))$

(x, y)

popular(y) := $\forall x \in P$ loves(x, y)

fluffier(x, y)   x is at least as fluffy as y

define <u>fluffiest(z)</u>

$\forall y \in P \wedge \exists z \; \exists p \; \text{fluffier}(z, y)$

?

$\forall y \in \text{Cats}, \; \text{fluffier}(x, \underline{y})$

$\exists y \in \text{Cats}, \; \text{fluffier}(x, y)$

$\neg(\exists y \in \text{Cats}, \; \text{fluffier}(y, x)))$

$\forall y \in Cats\ \exists x$ s.t. fluffier(x, y)

$\exists x$ fluffiest(x)

fluffiest (x) x is the fluffiest cat

(or tied for it)

fluffiest orange cat (x)

fluffiest of the orange cats

$$\frac{FOC(x) : x \in Cats}{Orange(x) \wedge fluffiest(x)}$$

is

$x \in Cats$ domain or type

$orange(x) \land \forall y \in Cats \; orange(y) \land fluffier(x, y)$

$FOC(x) : x \in Cats$

$orange(x) \land \forall y \in Cats \; orange(y) \rightarrow fluffier(x, y)$

P, professors S, students C, classes

$\text{ReqCourse}(c) = c \in C \; \forall s \in S \; \text{takes}(s, c)$

$\text{ProfofReq}(p): \exists c \in C : \text{ReqCourse}(c) \wedge \text{teaches}(p, c)$

$Q(p) \; \exists c \in C \; \forall s \in S \; (\text{takes}(s, c) \wedge \text{teaches}(p, c))$

there is a course that everyone takes that is taught by Prof p

$Q_2(p) \quad \forall s \in S \ \exists c \in C$

$(\text{takes}(s, c) \land \text{teacher}(p, c))$

all students take some course

that has this prof

Everyone climbs a tree

$\exists \forall$ ← there is one tree everyone climbs

$\forall \exists$ ← everyone climbs some tree but maybe not the same one

Theorems

A fully quantified expression of predicate logic is a theorem if and only if it is true for every possible interpretation of its predicates.

analogous to tautology