#Teaching

COSC 290: Discrete Structures

Relations, Functions, Countability

01/27/2026

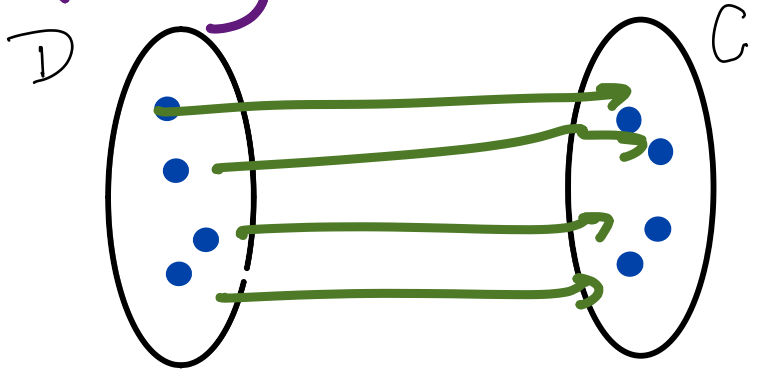
Prof. Forrest

Warm up & Goals

Warm up

1. Talk to your neighbor about the highlight of your snow day.

2. Consider the following domain, D, and codomain, C.



(a) give a relation that is one-to-one (ie injective) and not onto

(b) give a relation that is onto (ie surjective) and not one-to-one

(c) give a relation that is a one-to-one correspondence (ie bijective)

Logistics

- Codelet 1 is posted (due Friday)
- OH are R. 10:30–1:30
- notecards for in class problems
- Lab 1 Wed
- Codelet 1 use basic Python

Functions

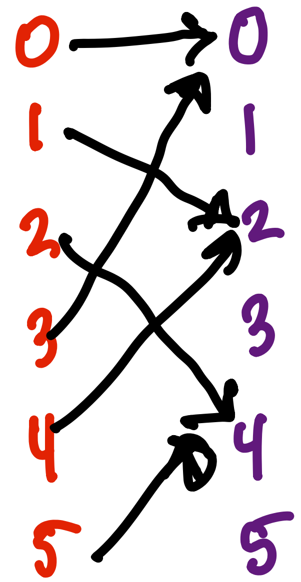
Let $f: \{0,1,2,3,4,5\} \to \{0,1,2,3,4,5\}$

$f(x) := 2x \bmod 6$

This function is

A) one-to-one
B) onto
C) bijective
D) none of the above

1



Let $g: \{0,1,2,\dots,9\} \to \{0,1,2,3,4\}$

$g(x) := \lfloor x/2 \rfloor \bmod 5$

This function is

A) one-to-one
B) onto
C) bijective
D) none of the above

2

$0, 1 \to 0$

$9/2$ 4 5

Let $h: \{0,1,2,3,4\} \to \{0,1,2,3,4\}$

$h(x) := 2x \bmod 5$

This function is

A) one-to-one
B) onto
C) bijective
D) none of the above

3

Cardinality Revisited

or, some infinities are built different

or, you can cram a shit ton, but not more, into infinity

Definition 1 The sets A and B have the same *cardinality* if and only if there is a one-to-one correspondence from A to B. When A and B have the same cardinality, we write $|A| = |B|$.

For infinite sets the definition of cardinality provides a relative measure of the sizes of two sets, rather than a measure of the size of one particular set. We can also define what it means for one set to have a smaller cardinality than another set.

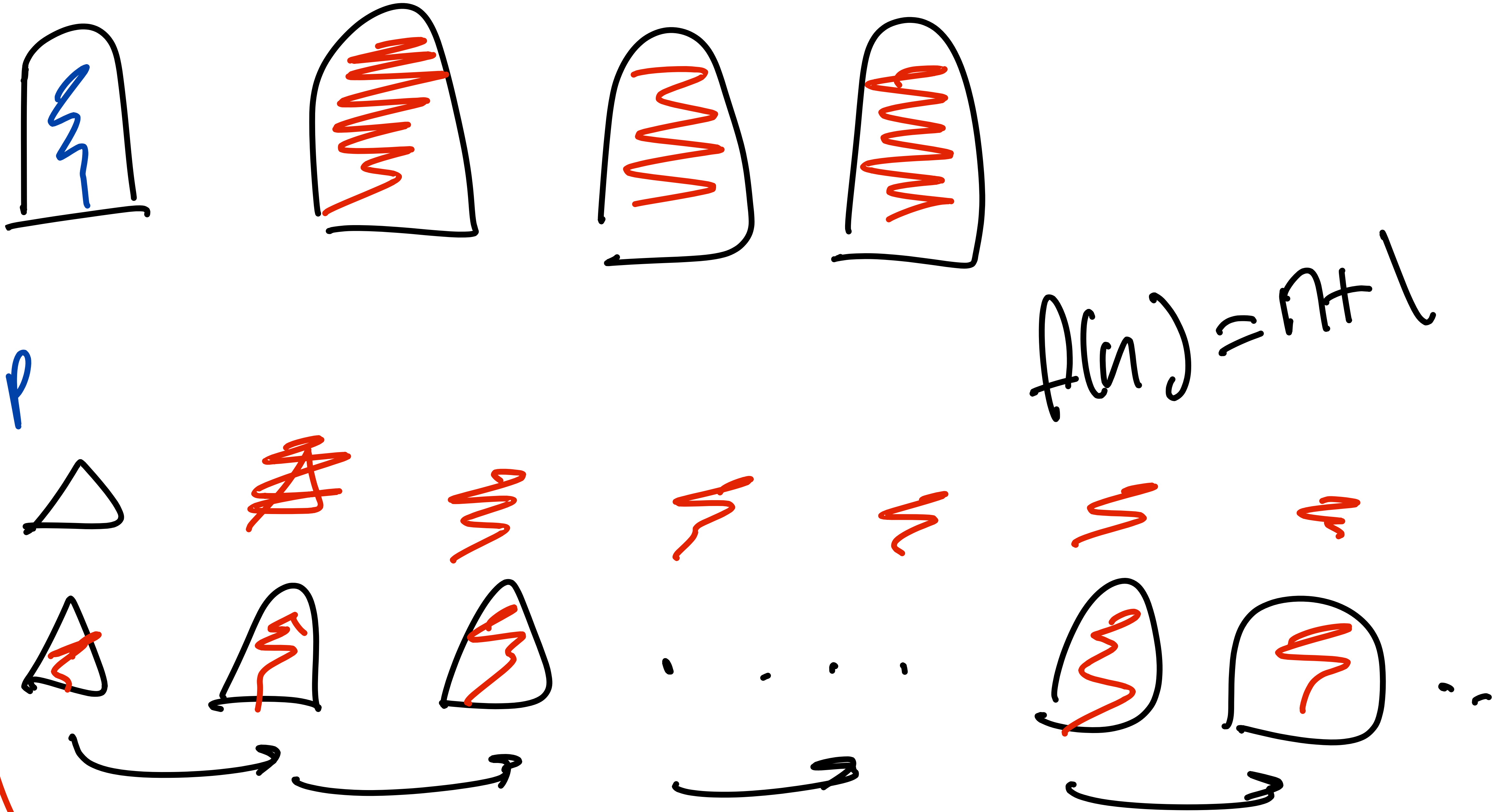
Definition 2 If there is a one-to-one function from A to B, the cardinality of A is less than or the same as the cardinality of B and we write $|A| \leq |B|$. Moreover, when $|A| \leq |B|$ and A and B have different cardinality, we say that the cardinality of A is less than the cardinality of B and we write $|A| < |B|$.

Countability

Definition 3 A set that is either finite or has the same cardinality as the set of positive integers is called *countable*. A set that is not countable is called *uncountable*. When an infinite set S is countable, we denote the cardinality of S by $\aleph_0$ (where $\aleph$ is aleph, the first letter of the Hebrew alphabet). We write $|S| = \aleph_0$ and say that S has cardinality "aleph null."

$$N_0 = \{1, 2, 3, 4 \cdots\cdots \infty\}$$

Finite Hotels



1

2 walk up

$f(n) = n+1$

all full

a person arrives

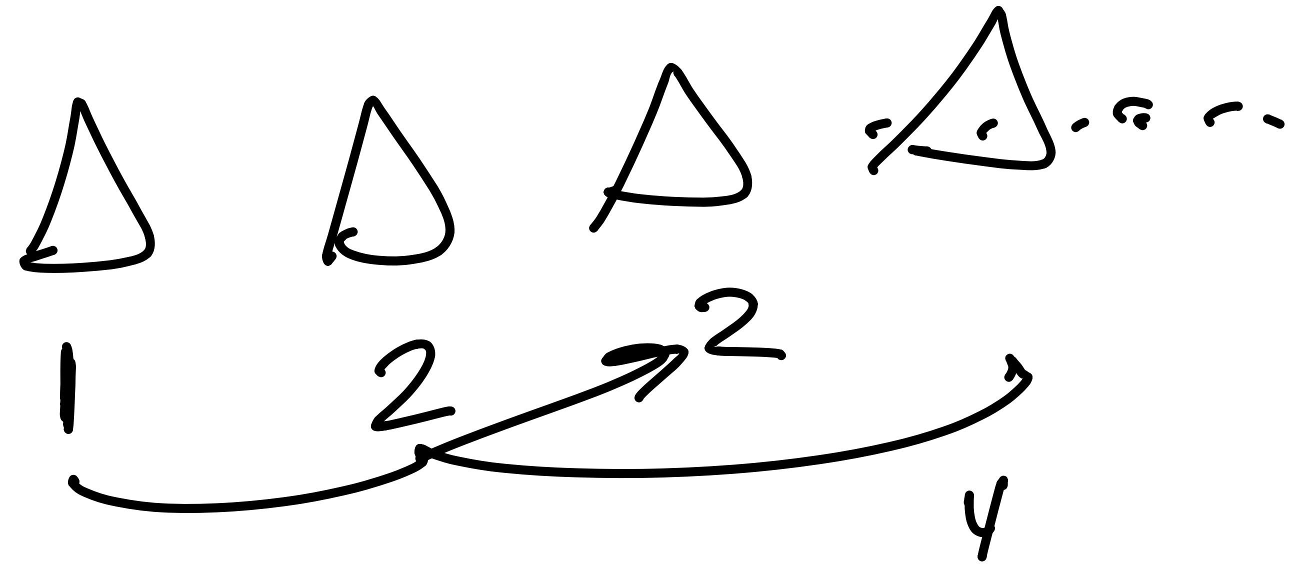
$\boxed{\text{Can I fit them in the hotel?}}$

5 people ?

how do we fit five more people

$f(n) = 2n$

$f(n) = n+5$



$\infty = \infty + 5$

1, 2 . . . . . .

n

$\infty = \infty + \infty$

positive number $\mathbb{N}$, $= \{1, 2, \cdots\}$

integers $= \{\cdots -2\ -1\ 0\ 1\ 2 \cdots\}$

$0\ -1\ 1\ -2\ 2\ -3\ 3\ \cdots$

$1\ 2\ 3\ 4\ \cdots$

Infinite Hotels

rational numbers? $\mathbb{Q}$

$\frac{1}{2}$, $1/3$, $1/16$, $2/3$, $2/20$ . . . . .

are the rational numbers countable?

1/1	1/2	1/3	1/4	1/5	…	∞
2/1	2/2	2/3	2/4	2/5	…	
3/1	3/2					
4/1	4/2					
5/1	5/2					
⋮	⋮					
∞						

$|Q| = |N_0|$

order p/q

$p+q$

break ties by $p > q$

$2 = 1/1$

$3 = 1/2,\ 2/1$

$4 = 1/3,\ 2/2,\ 3/1$

⋮

there exists sets that are not countable

$S = \{ \text{decimal expansions of only } 3\text{s and } 4\text{s} \}$

$x_1 = 0.343443 \ldots$

$x_2 = 0.44434443 \ldots$

$x_3 = 0.3343344 \ldots$

$x_4 = \ldots$

1. Assume a bijection $f: \mathbb{N}_0 \to S$
2. find an element of S that is not in $f(n)$
3. f is no surjective
4. f is not a bijective

$S = \{$ decimal expansions of only ... $\}$

$x_1 = 0.34\,3443 \ldots$

$x_2 = 0.44434443 \ldots$

$x_3 = 0.334344 \ldots$

$x_4 = \ldots$

$x_5 = 0.333334$

$x_s = 0.433\ldots4$

$x_s \notin f(S)$

$\mathbb{R}$ is not countable

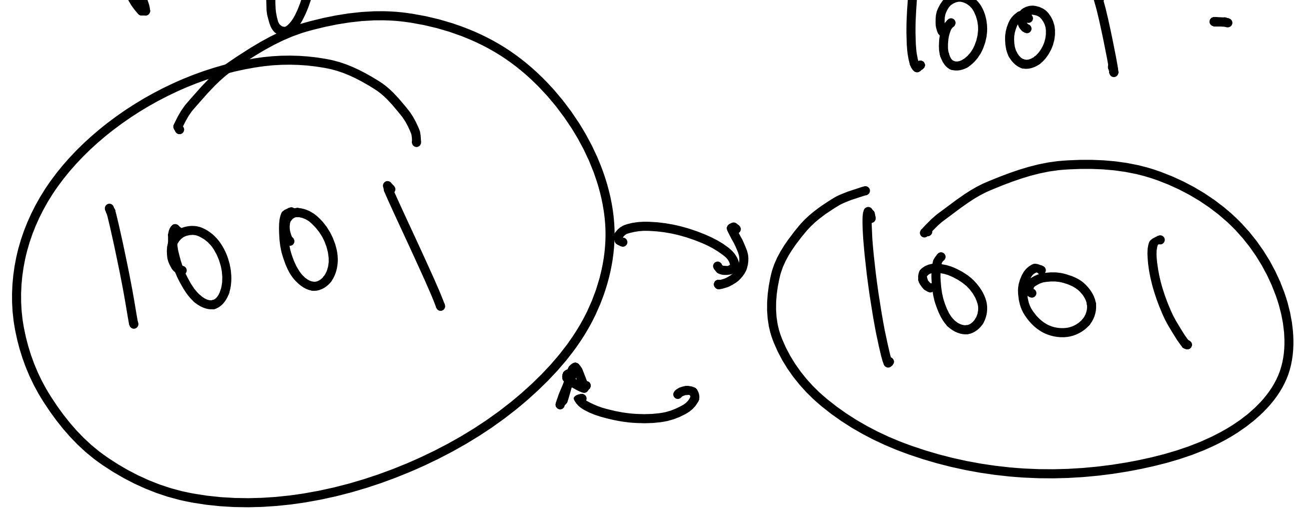
$0.12 \ldots$

Your Turn

Demonstrate that the set of all computer programs is infinite and countable

programs → binary → convert base 10

1001 . . . . .



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Propositional Logic I

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Propositional Logic Formulas

Which of these statements is a proposition?

(a) $x+1=3$ ✗

(b) $1+4=7$ False ✓

(c) Colgate is in Hamilton, NY. T

(d) Answer this question. ✗

Propositions are True or False

atoms

$p, q, r, s \dots$

$\neg p$

$p \wedge q$

connectives

$\wedge$ (and)

$\vee$ (or)

$\rightarrow$ (implies)

$\oplus$ (xor)

Truth Tables

p	q	$p \wedge q$	$p \vee q$	$p \oplus q$
T	T	T	T	F
T	F	F	T	T
F	T	F	T	T
F	F	F	F	F

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

if $\overbrace{\text{it snows tomorrow}}^{p}$, then class will be canceled

q

$p \to q$

T

Give the truth table for $(p \vee q) \rightarrow r$

if my car dies
or
if snows tomorrow
then class is cancelled

how do I show this is a lie?

2^3

p	q	r	$p \vee q$	$(p \vee q) \rightarrow r$
T	T	T	T	
T	F	T	T	
T	T	F	T	F
T	F	F	T	F
F	T	T	T	
F	F	T	F	
F	T	F	T	F
F	F	F	F	

Complete a Notecard