#Teaching

COSC 290: Discrete Structures

Relations, Functions, Countability

01/22/2026

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Warm up & Goals

Warm up

1. Talk to your neighbor about your favorite holiday dessert.

2. Use the set abstraction (or set builder) notation to characterize each of these sets:

(a) $\{0,3,6,9,12\}$ $\quad \{3x \mid x \in \mathbb{N},\ 0 \le x \le 4\}$

0, 1, 2, ...

(b) $\{-3,-2,-1,0,1,2,3\}$ $\quad \{x \mid -3 \le x \le 3,\ x \in \mathbb{Z}\}$

all integers

(c) $\{$"n", "e", "o", "p"$\}$ $\quad \{x \mid x$ in "nope"$\}$

Logistics

- Codelet 1 is posted (due next Friday)
- OH are Rs 10:30–1:30
- notecards for in class problems

Learning objectives

- Describe the move from sets to relations to functions
- Describe injections, surjections, and bijections
- Articulate what a countable set is
- Understand a limit to computation

Reviewing Sets

Sets must have unique elements. If there are duplicates then that set is a multiset

Frequent Itemset Mining

Assume we have itemsets n I: $I_1\ I_2 \cdots I_n$ where each itemset contains purchased items.

$I_1 = \{\text{coffee, cat food}\}$

$I_2 = \{\text{coffee, soy milk, tofu}\}$

$I_3 = \{\text{soy milk, choco puffs, Coffee}\}$

$I_4 = \{\text{beer}\}$

$\vdots$

We defined the support for a potential itemset J to be the number of itemsets I where $J \subseteq I$

potential set

threshold

```
def SetIsFreq(J, θ):
    count = 0
    for i in 1 ... n:
        if J ⊆ I_i:
            count += 1
    if count/n > θ:
        return True
    return False
```

How do we find all the frequent itemsets?

$U_{\text{all items}} = I_1 \cup I_2 \cdots \cup I_n$

```
Freqs = { }
for J in P(U):
    if SetIsFreq(J, 0.01):
        Freqs = Freqs ∪ {J}
```

$O(2^{|U|})$

How might we speed this up (at least in practice)?

{advil} {water} {coffee}

frequent single sets

3 sets multiple

frequent doubles

⋮

$J = \{\text{coffee, advil}\}$

0.001%

$J' = \{\text{coffee, advil, water}\}$

Computer science studies computable objects and all computable objects are countable.

Sets vs. Sequences

$A = \{3, 1\}$ $B = \{1, 3\}$

$A = B$!!

$A = \langle 3, 1 \rangle$ $B = \langle 1, 3 \rangle$

$A \neq B$

$A \times B = \{ \langle a, b \rangle \mid a \in A \text{ and } b \in B \}$

$A = \{3, 1\}$ $B = \{1, 2, 3\}$

what is $A \times B$

$A = \{3, 1\}$ $B = \{1, 2, 3\}$

what is $A \times B$

① $\langle 3, 1, 2 \rangle$

② $\{ \langle 3, 1 \rangle, \langle 3, 2 \rangle, \langle 3, 3 \rangle,$
$\langle 1, 1 \rangle, \langle 1, 2 \rangle, \langle 1, 3 \rangle \}$

$\langle 2, 1 \rangle$

$A := \{\text{Colgate}, \text{'Gate}\}$

$B := \{\text{raiders}, \text{university}, \text{hockey}\}$

$C := \{!\}$

$$(A \times B) \times C \stackrel{?}{=} A \times (B \times C)$$

$$\langle \langle \text{colgate}, \text{raiders} \rangle, ! \rangle$$

$$\mathbb{R}^3$$

$$\langle \text{colgate}, \text{raiders}, ! \rangle$$

Application

traceroute

when you connect to a website you are routed through other ip addresses.

Let's say that traceroute returns a path from your IP address to a target IP address. This path is at most 10 steps.

How would you define the set of all possible traceroutes?

$S = \{\langle IP_1 \rangle, \langle IP_1, IP_2 \rangle \ldots$
$\langle IP1, IP_2 \ldots, IP_{10} \rangle\}$

$\{x \mid 1 \leq x \leq 10$ and $x \in \mathbb{Z}$ and in $S\}$

$\text{YourIP} \times \mathcal{P}(S)$

$S = \{$ip addresses $\ldots\}$

$S^1 \cup S^2 \cup S^3 \cup S^4 \cup \ldots$

S^{10}

Relations (Basic)

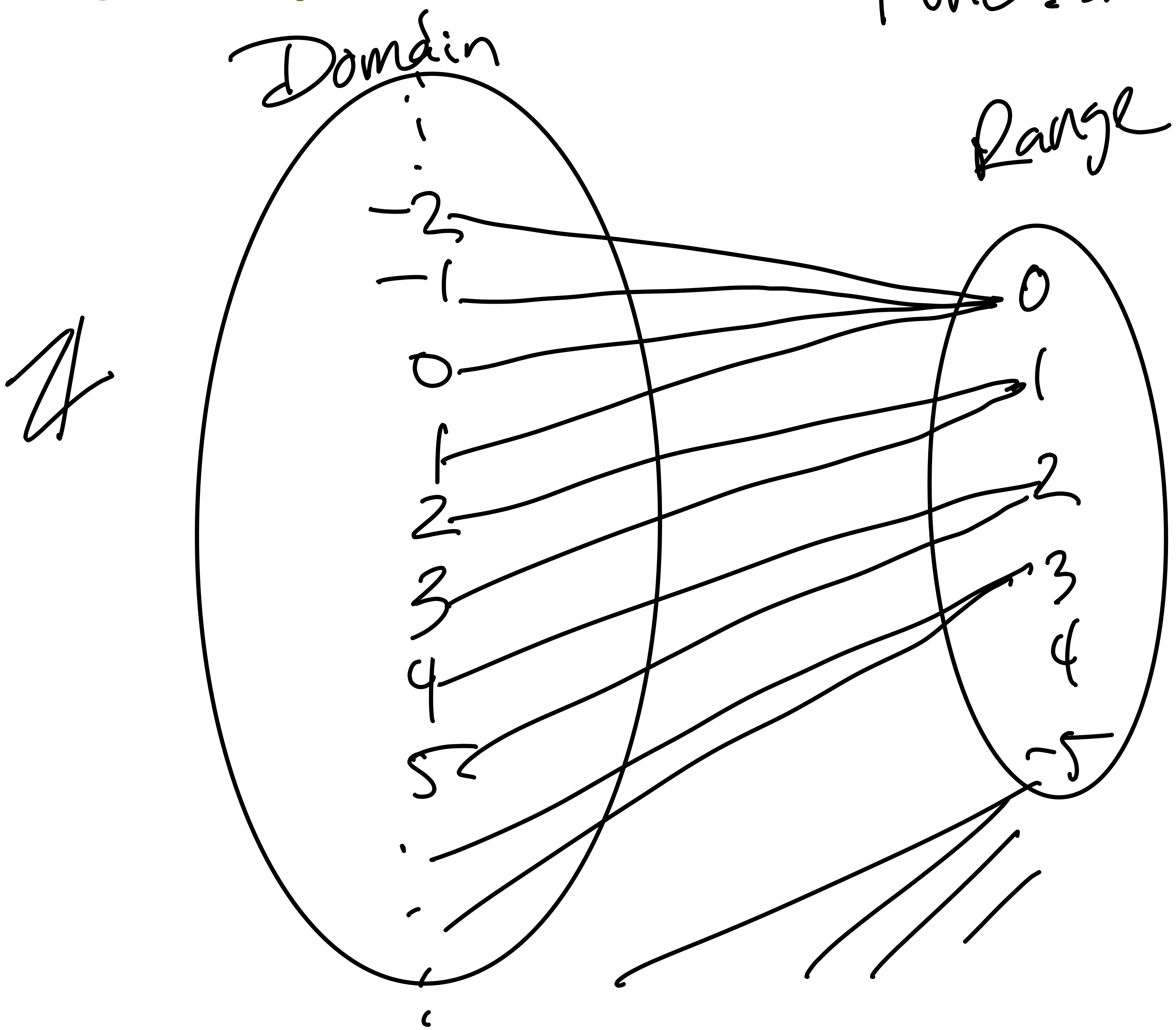
what is the set representing the domain of func?

what is the set representing the range (image) of func?

```
def func(x: int):
    if x < 0:
        return 0
    if x > 9:
        return -5
    return x // 2
```

$\langle -2, 0 \rangle$

$func : \mathbb{Z} \rightarrow \{0, 1, 2, 3, 4, -5\}$



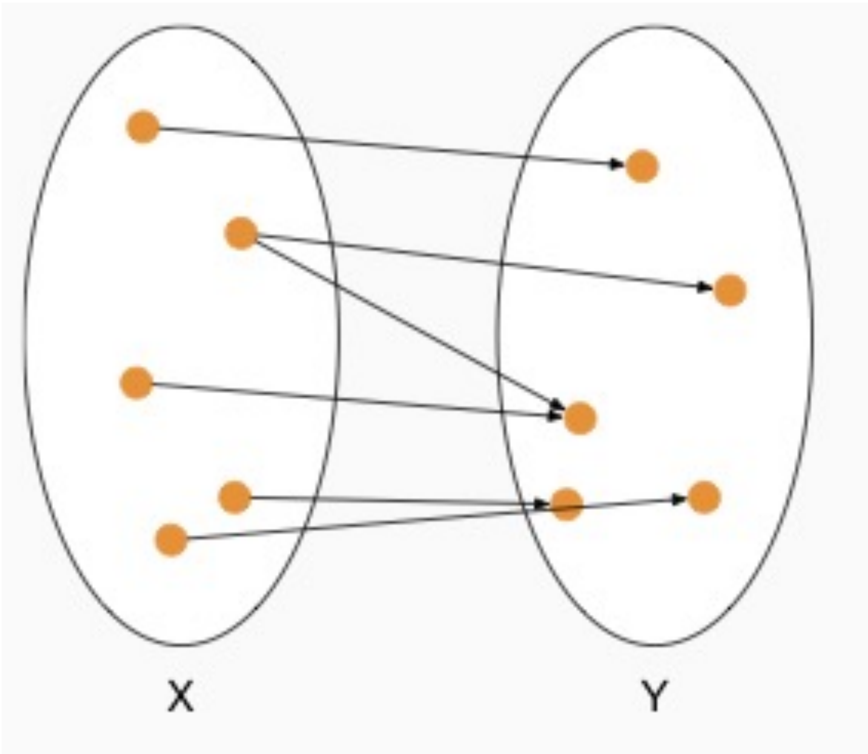
Functions

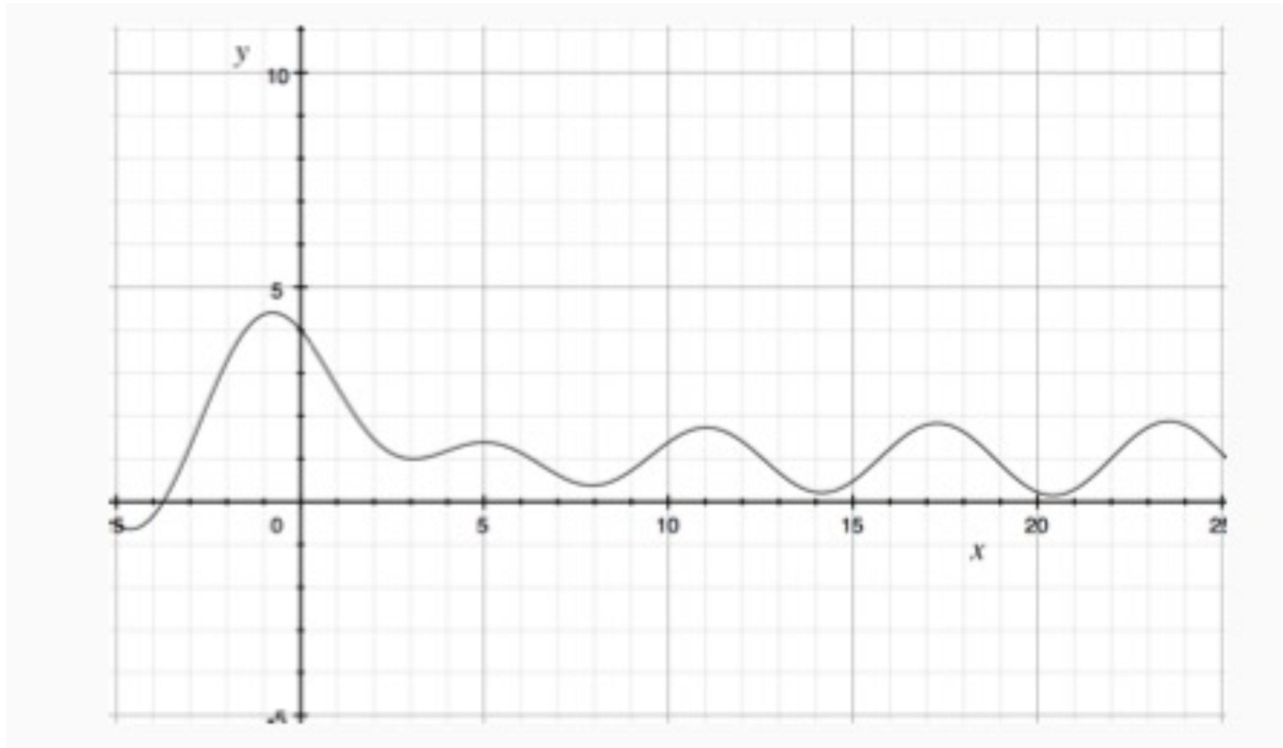
$f: A \to B$ or $f \subseteq A \times B$

C) $X \to Y$

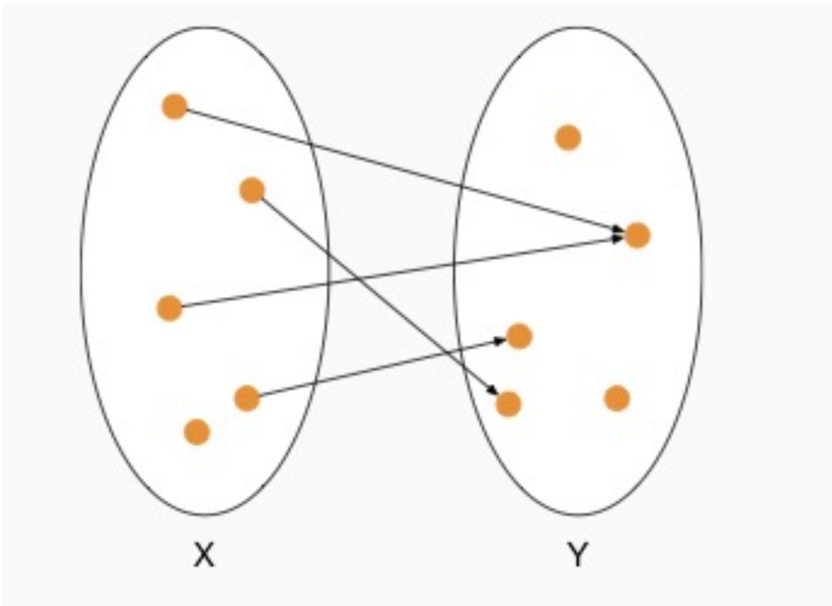
D) $Y \to X$

A)





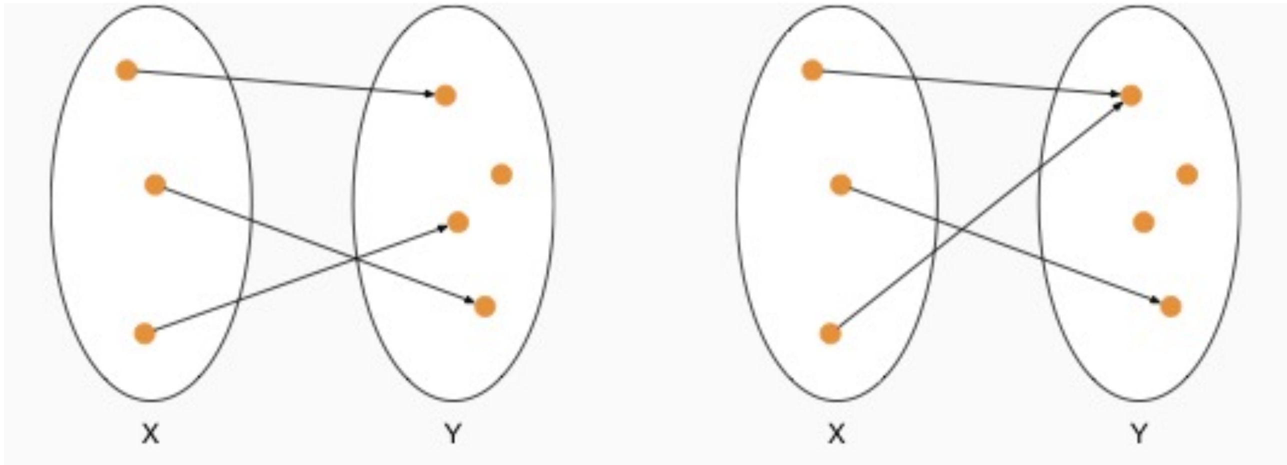
B)



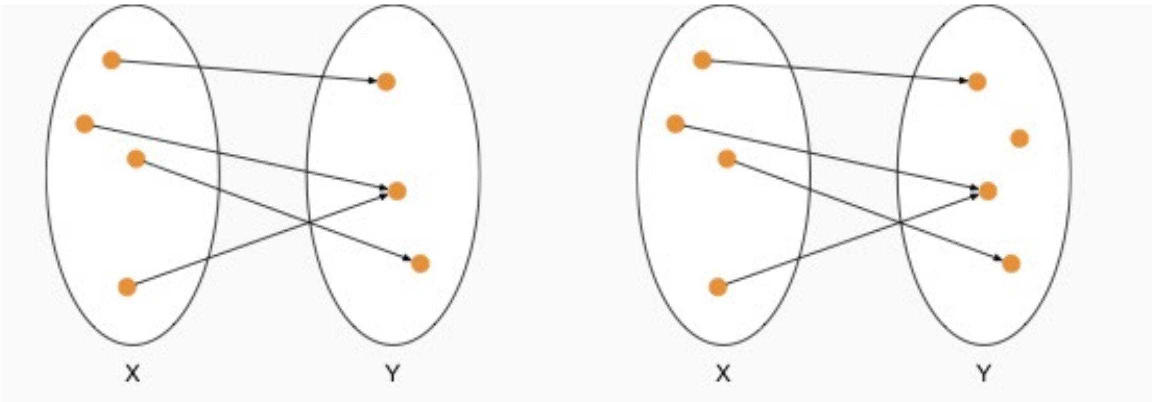
Are A, B, C, and D functions?

Functions

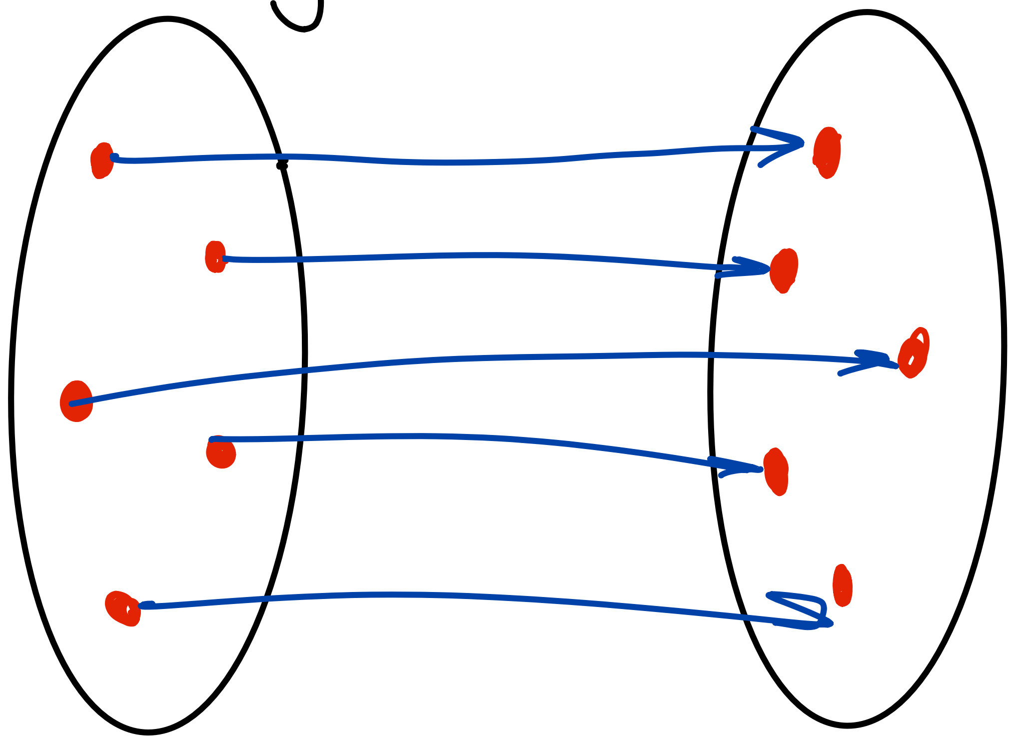
One-to-one?



onto?



bijective?



Functions

Let $f : \{0,1,2,3,4,5\} \longrightarrow \{0,1,2,3,4,5\}$

$$f(x) := 2x \bmod 6$$

This function is

A) one-to-one

B) onto

C) bijective

D) none of the above

1

Let $g : \{0,1,2,\dots,9\} \longrightarrow \{0,1,2,3,4$

$$g(x) := \lfloor x/2 \rfloor \bmod 5$$

This function is

A) one-to-one

B) onto

C) bijective

D) none of the above

2

Let $h : \{0,1,2,3,4\} \longrightarrow \{0,1,2,3,4\}$

$$h(x) := 2x \bmod 5$$

This function is

A) one-to-one

B) onto

C) bijective

D) none of the above

3

Cardinality Revisited

or, some infinities are built different

or, you can cram a shit ton, but not more, into infinity

Definition 1 The sets A and B have the same *cardinality* if and only if there is a one-to-one correspondence from A to B. When A and B have the same cardinality, we write $|A| = |B|$.

For infinite sets the definition of cardinality provides a relative measure of the sizes of two sets, rather than a measure of the size of one particular set. We can also define what it means for one set to have a smaller cardinality than another set.

Definition 2 If there is a one-to-one function from A to B, the cardinality of A is less than or the same as the cardinality of B and we write $|A| \leq |B|$. Moreover, when $|A| \leq |B|$ and A and B have different cardinality, we say that the cardinality of A is less than the cardinality of B and we write $|A| < |B|$.

Countability

Definition 3 A set that is either finite or has the same cardinality as the set of positive integers is called *countable*. A set that is not countable is called *uncountable*. When an infinite set S is countable, we denote the cardinality of S by $\aleph_0$ (where $\aleph$ is aleph, the first letter of the Hebrew alphabet). We write $|S| = \aleph_0$ and say that S has cardinality "aleph null."

$1 \quad 2 \quad 3 \quad \cdots \quad \infty$

$0 \quad 1 \quad 2 \quad 3 \quad \cdots \quad \mathbb{N}$

$1 \quad 2 \quad 3 \quad \cdots \quad \mathbb{N}^{>0}$

$$f(n) = n+1$$

$$
\begin{array}{cccccccc}
O & = & 1, & 3, & 5, & 9, & \cdots \\
 & & \updownarrow & \updownarrow & \updownarrow & \updownarrow & \\
 & & 1 & 2 & 3 & 4 & \cdots
\end{array}
$$

$$|O| = |\mathbb{N}|$$

$$f(n) = ?$$