Sets and Functions

COSC 290: Discrete Structures
Spring 2026
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Warm-up

1. Talk to the person next to about whether hot cider or hot chocolate is the better winter drink
2. Consider the following code snippet in Python. What is its worst case runtime?

```
def find_char(word: str, char: str) -> bool:
""" Returns whether the string has char in it

Args:
    word (str): A string of characters
    char (str): A single character
Returns:
    (bool): True if char is in word, else False
"""
for c in word:
    if c == char:
        return True
return False
```

Logistics

- No lab this week

Learning Objectives

- Recognize basic terminology
- Instantiate sets and apply operations to them
- Apply basic operations to matrices

Important Sets and Basic Terms

- $\mathbb{N} = \{0, 1, 2, 3, \dots\}$, the set of all **natural numbers**
- $\mathbb{Z} = \{\dots, -2, -1, 0, 1, 2, \dots\}$, the set of all **integers**
- $\mathbb{Z}^+ = \{1, 2, 3, \dots\}$, the set of all **positive integers**
- $\mathbb{Q} = \{p/q \mid p \in \mathbb{Z}, q \in \mathbb{Z}, \text{ and } q \neq 0\}$, the set of all **rational numbers**
- $\mathbb{R}$, the set of all **real numbers**
- $\mathbb{R}^+$, the set of all **positive real numbers**
- $\mathbb{C}$, the set of all **complex numbers**
- Modulus: if $x \mod 2 = 0$, then x is $\dots$?

Comprehension Problems

Question 1

Give the abstraction definition of the set $\{1, 3, 5, 7, 9\}$

Question 2

Which sets are equal?

$$R \coloneqq \{1+1, 2+2, 3+3, 4+4\}$$
$$S \coloneqq \{8, 4, 8, 2, 6, 4\}$$
$$T \coloneqq \{2, 4, 6, 8\}$$

Question 3

Which sets are equal?

$$R := \{2x : x \in \mathbb{Z}^{>0} \text{ and } x < 10\}$$
$$S := \{x \in \mathbb{Z}^{>0} : x \bmod 2 = 0 \text{ and } x < 10\}$$
$$T := \{2, 4, 6, 8\}$$

Question 4

Find the sets A and B if $A - B = \{1, 5, 7, 8\}$, $B - A = \{2, 10\}$, and $A \cap B = \{3, 6, 9\}$.

Question 5

Let A and B be two sets with $|A| = m$ and $|B| = n$ and and $m < n$. What is the **smallest** cardinality for $A \cup B$? In other words, what must $|A \cup B|$ be at least?

Question 6

What is the power set of $\{cat, dog, lizard\}$?

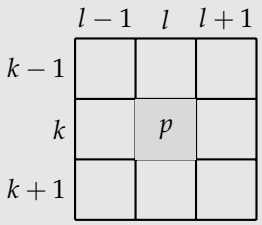
Generalization Problems

Practice 1

Propose an algorithm for Frequent Itemset Mining. A **frequent itemset** is a collection of items that are frequently purchased together (by at least 1% of customers, for example).

Practice 2

Suppose we want to apply a blur filter to an image. This blur filter replaces pixel p with the average of all pixels with ± 1 rows and columns of p. In this figure, pixel p is in row k and column l.



Let $pix(x, y)$ be a function that takes in a row x and column y and returns the *current* pixel value at x, y. **Write an expression that uses summations for the *desired* value for pixel k, l.**

Challenge Generalize to $\pm w$ rows.

Before Next Class

- Complete the course survey
- Read the selected parts of the book